Lecture 6

Cryptography 2: square attacks & prng

$whoami

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- CERT Polska / p4 ctf
- Malware researcher
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- Dekryptory do ransomware, łamanie krypto innych ludzi, testy bezpieczeństwa, CTF
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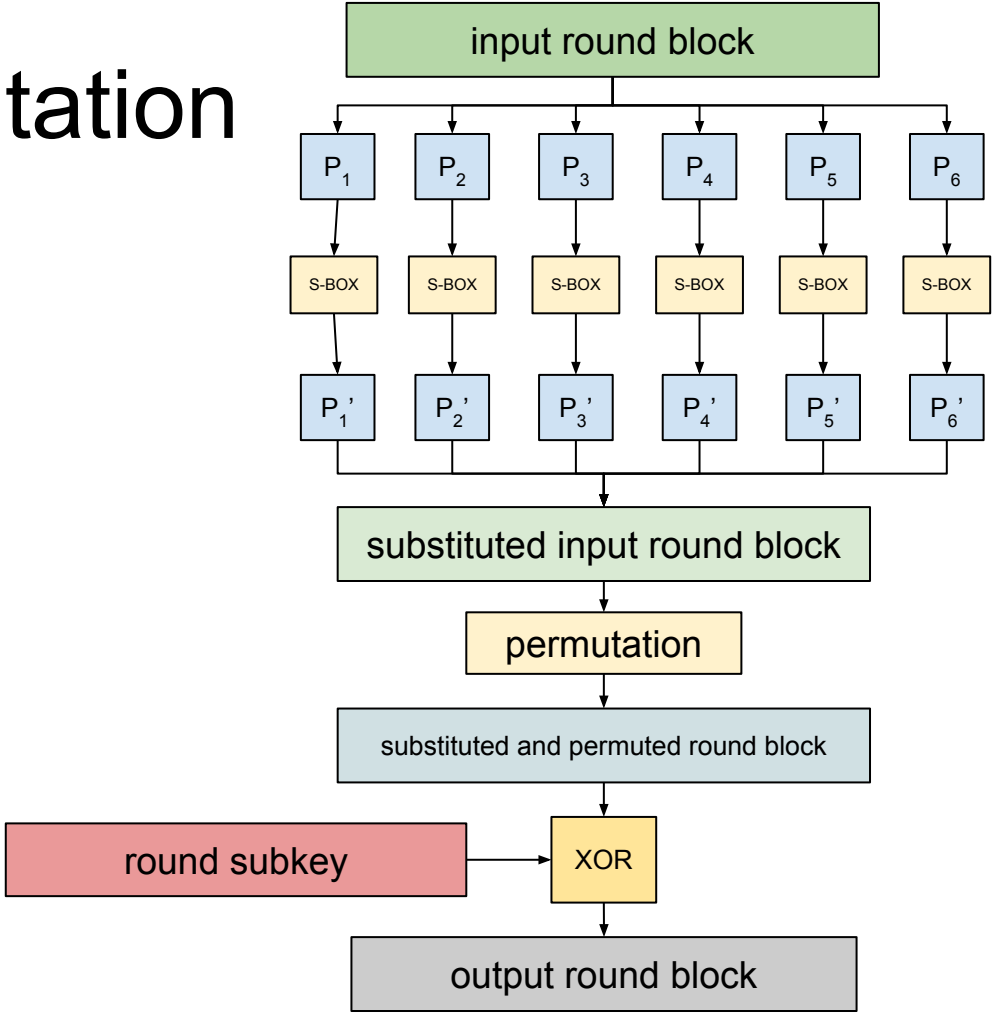
- crypto, re, algo, web, low-level

Substitution-permutation ciphers

- Square attack

Substitution-permutation ciphers

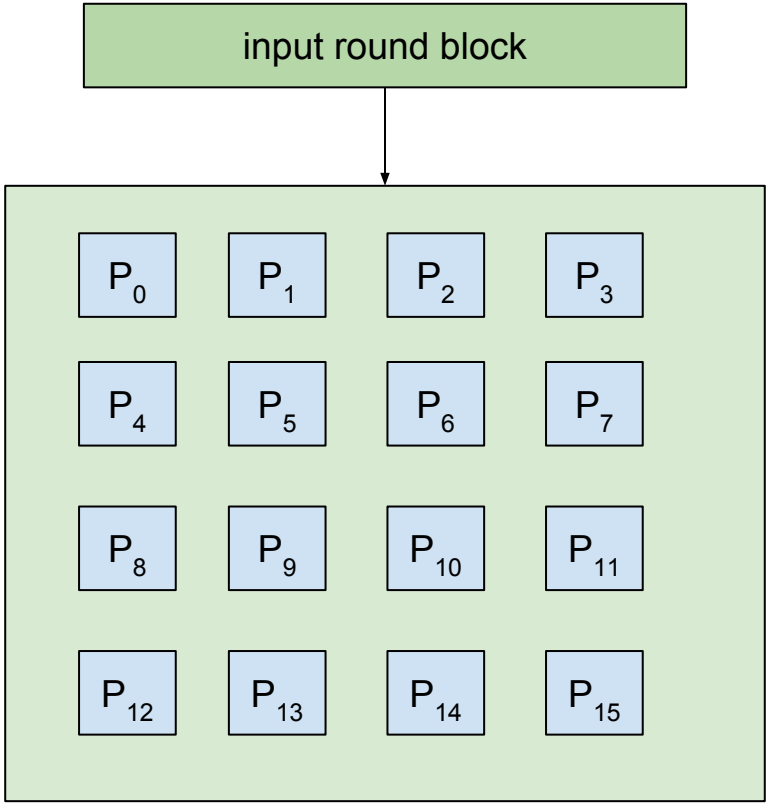
- S-Box
- P-Box
- Combine with key
- Confusion and diffusion



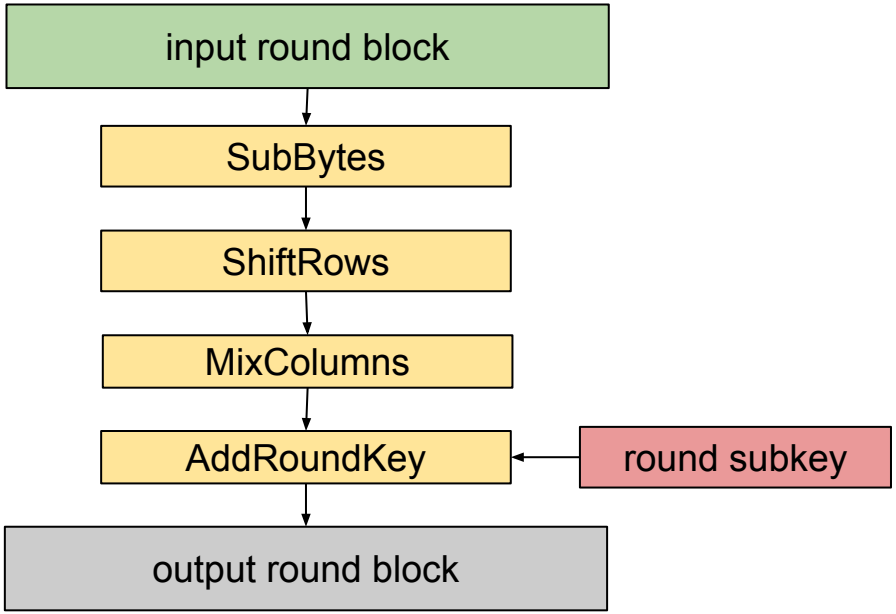
Square attacks

- Integral cryptanalysis
- AES (reduced to 4 rounds (from 10))
- Chosen plaintext

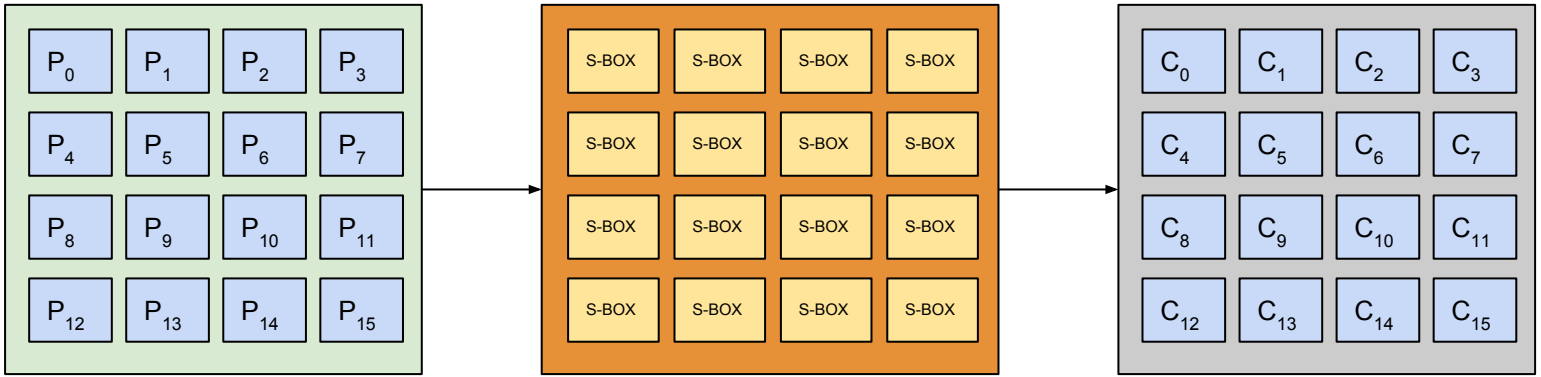
Square


input round block
P_0
P_1
P_2
P_3
P_4
P_5
P_6
P_7
P_8
P_9
P_{10}
P_{11}
P_{12}
P_{13}
P_{14}
P_{15}

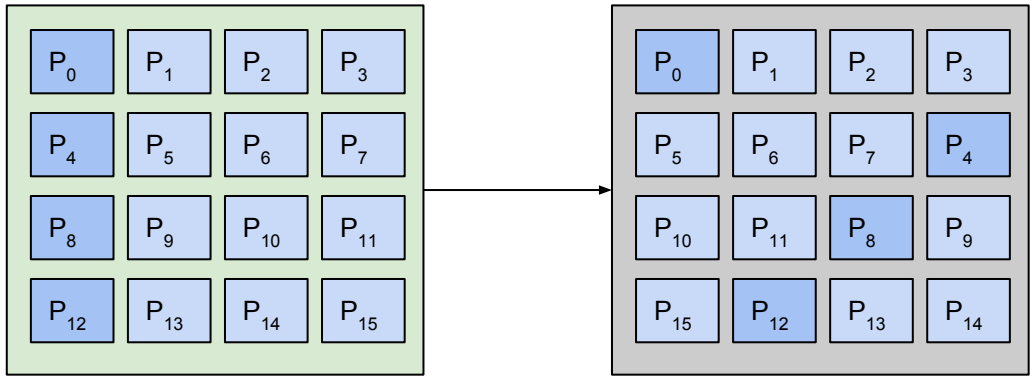
AES


input round block
SubBytes
ShiftRows
MixColumns
AddRoundKey
round subkey
output round block

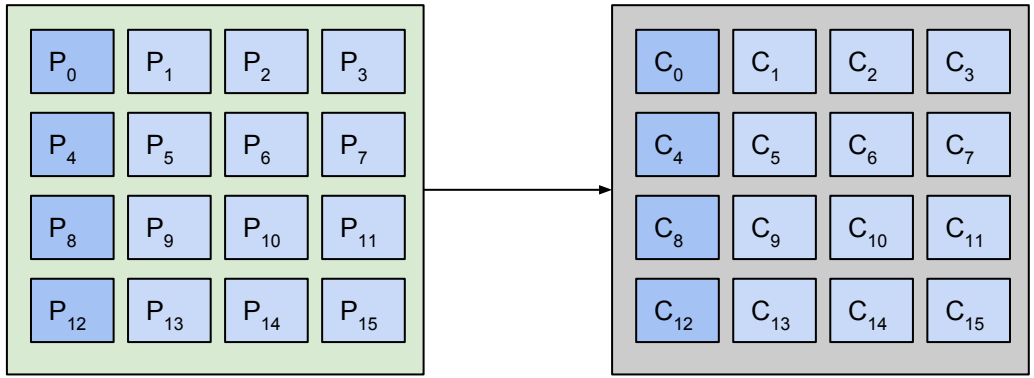
SubBytes


P0 P1 P2 P3
P4 P5 P6 P7
P8 P9 P10 P11
P12 P13 P14 P15
S-BOX S-BOX S-BOX S-BOX
S-BOX S-BOX S-BOX S-BOX
S-BOX S-BOX S-BOX S-BOX
S-BOX S-BOX S-BOX S-BOX
C0 C1 C2 C3
C4 C5 C6 C7
C8 C9 C10 C11
C12 C13 C14 C15

ShiftRows


P0 P1 P2 P3
P4 P5 P6 P7
P8 P9 P10 P11
P12 P13 P14 P15
P0 P1 P2 P3
P5 P6 P7 P4
P10 P11 P8 P9
P15 P12 P13 P14

MixColumns



$GF(2^8)$, $p(x) = P_{12}x^3+P_8x^2+P_4x+P_0$, $a(x) = 3x^3+x^2+x+2$, $p(x) * a(x)$, mod x^4+1

$C_0 = 2P_0 \oplus 3P_4 \oplus P_8 \oplus P_{12}$

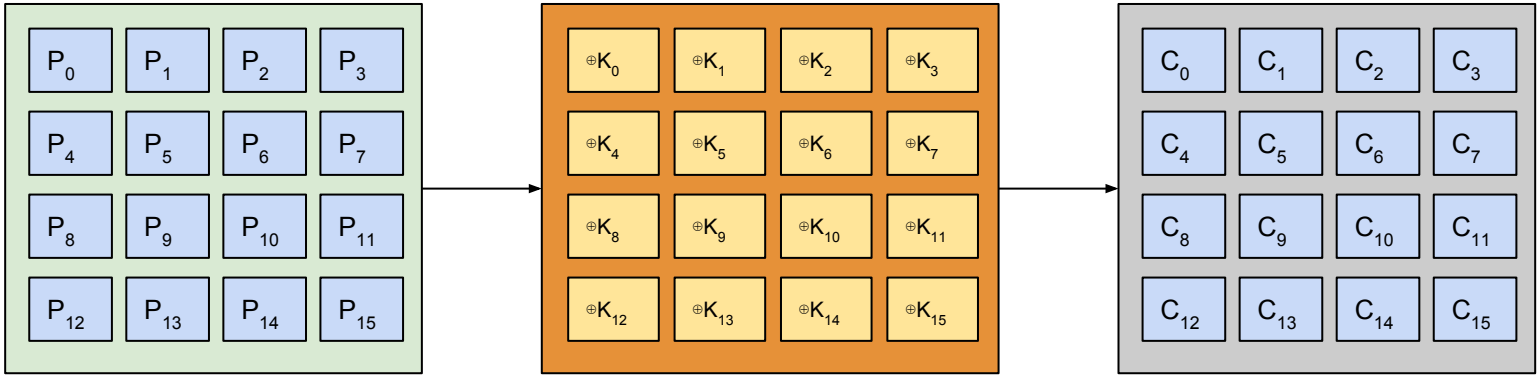
$C_4 = 2P_4 \oplus 3P_8 \oplus P_{12} \oplus P_0$

$C_8 = 2P_8 \oplus 3P_{12} \oplus P_0 \oplus P_4$

$C_{12} = 2P_{12} \oplus 3P_0 \oplus P_4 \oplus P_3$

Not present in last round

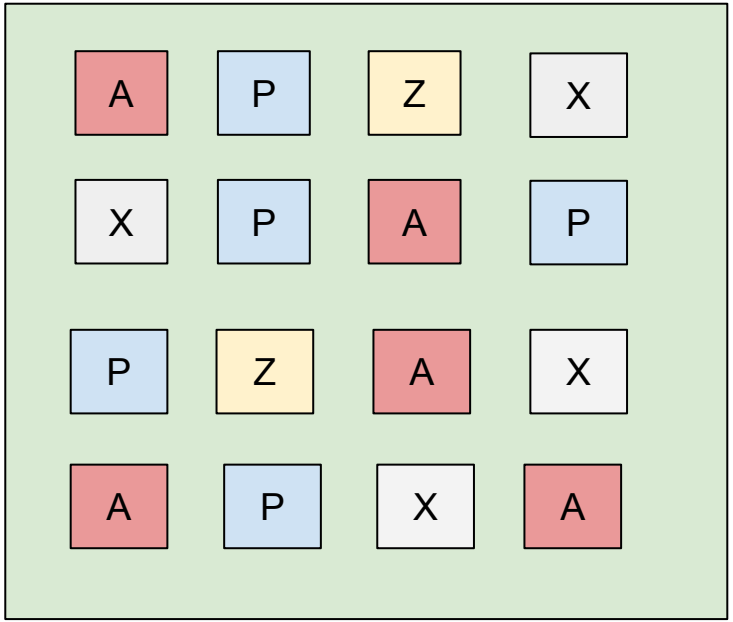
AddRoundKey



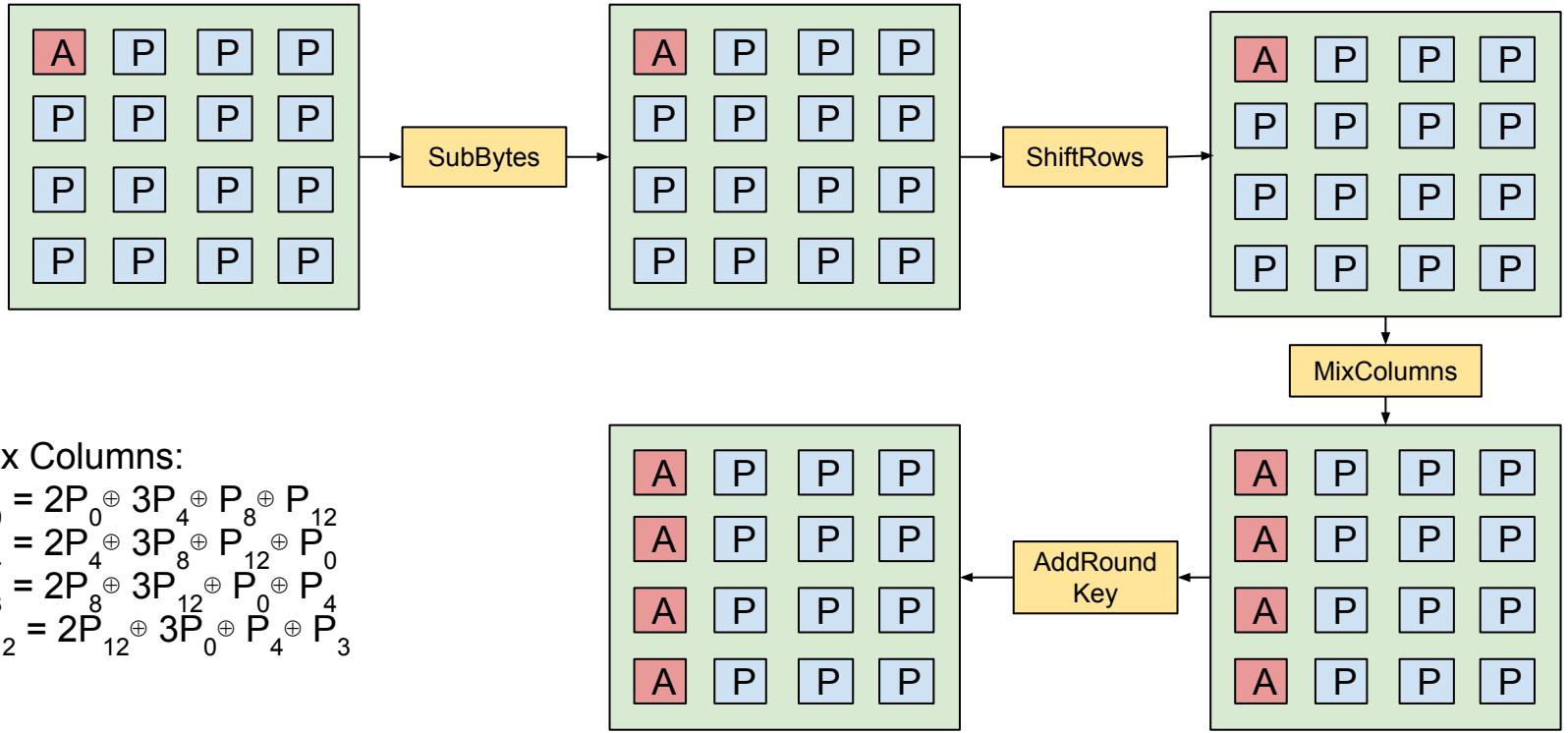
Active and passive byte states

- Set of plaintext
- Passive state: for each two plaintext the same square element
- Active state: for each two plaintext different square element
- Zero state: $\oplus$ elements from all plaintext = 0
  - $0 \oplus 1 \oplus ... \oplus 255 = 0$

- Example:
  - 256 plaintext, first byte: 0,1,2,..,255, other bytes: 0
  - Before first round: top left element is active, other elements passive



Round 1



Mix Columns:

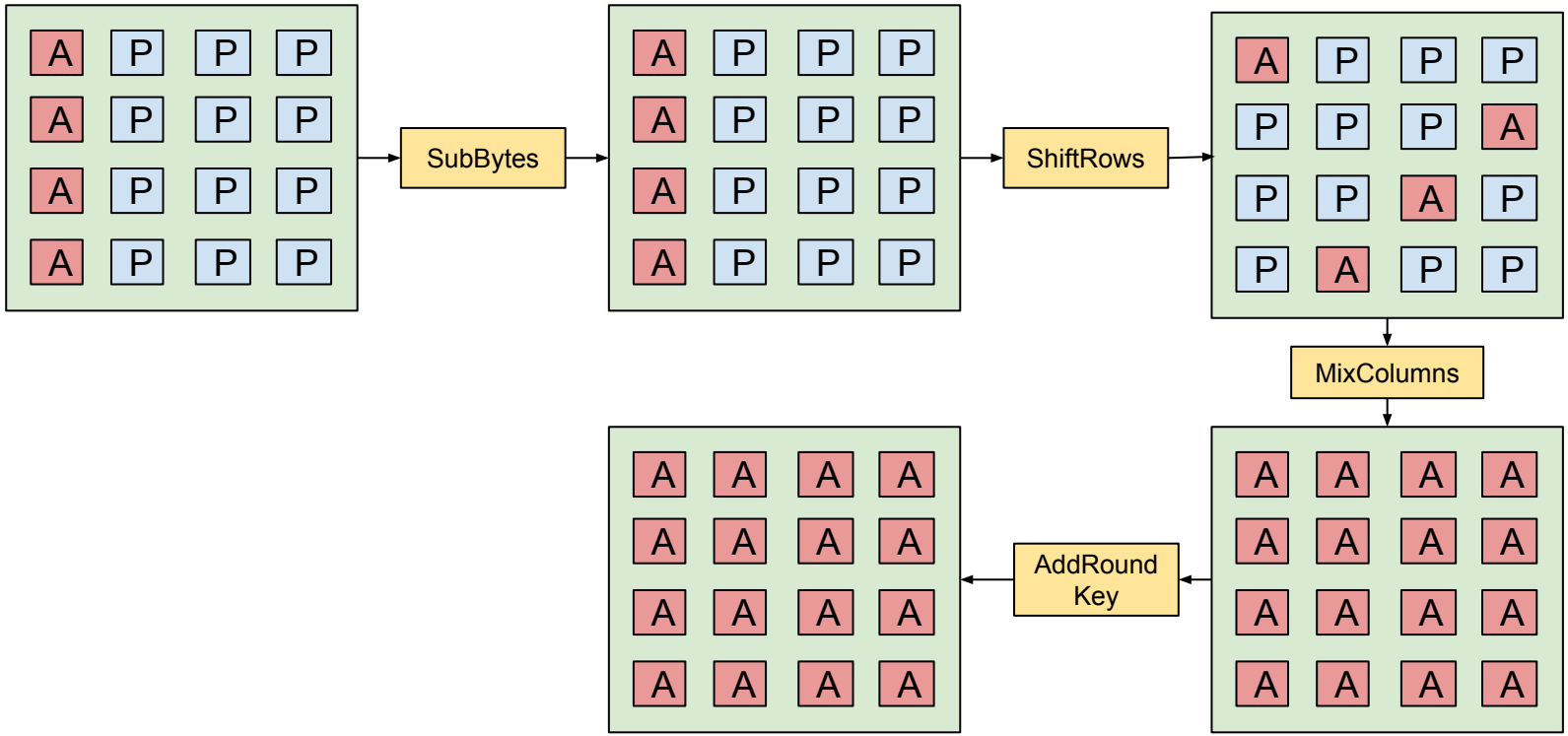
$C_0 = 2P_0 \oplus 3P_4 \oplus P_8 \oplus P_{12}$

$C_4 = 2P_4 \oplus 3P_8 \oplus P_{12} \oplus P_0$

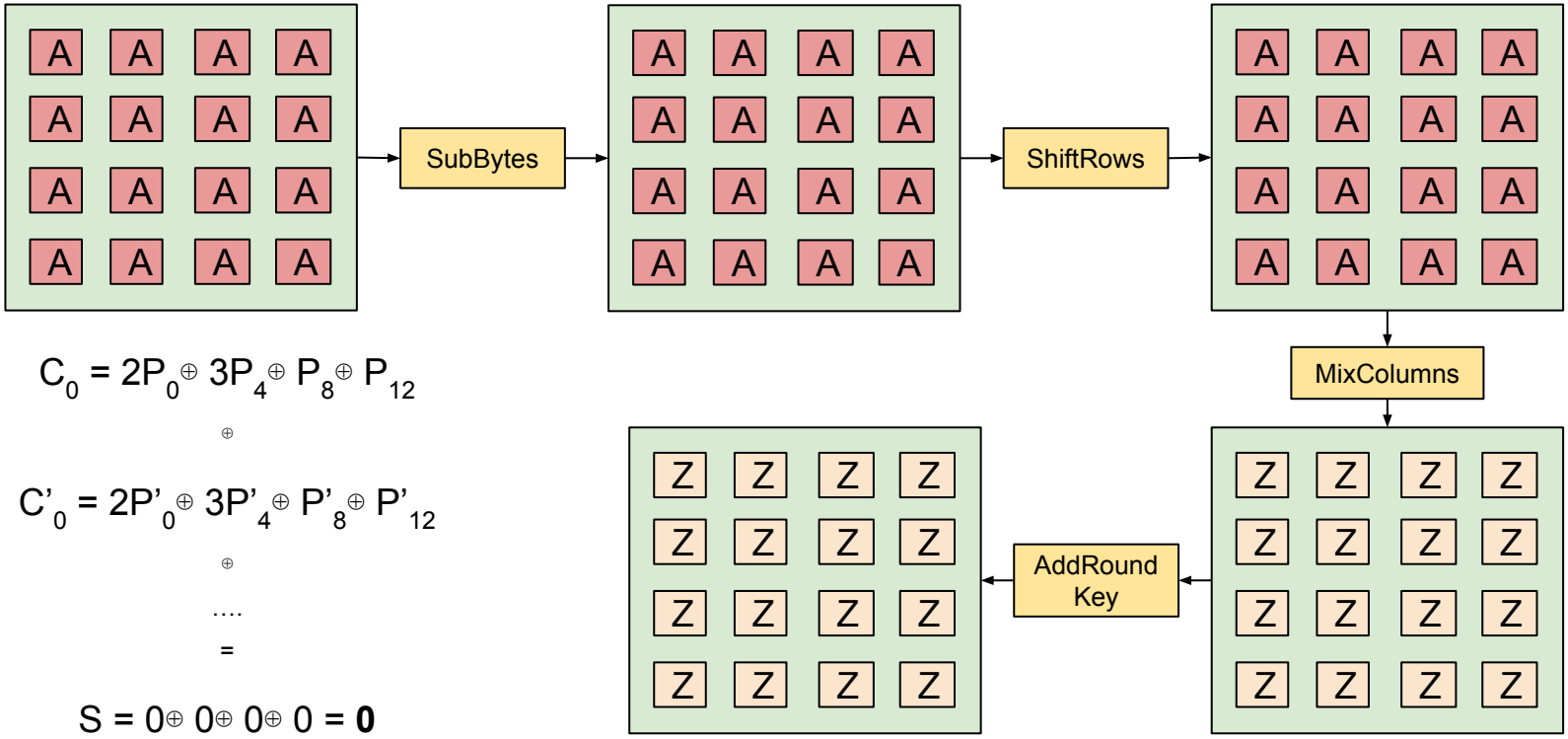
$C_8 = 2P_8 \oplus 3P_{12} \oplus P_0 \oplus P_4$

$C_{12} = 2P_{12} \oplus 3P_0 \oplus P_4 \oplus P_3$

Round 2



Round 3



$C_0 = 2P_0 \oplus 3P_4 \oplus P_8 \oplus P_{12}$

$\oplus$

$C'_0 = 2P'_0 \oplus 3P'_4 \oplus P'_8 \oplus P'_{12}$

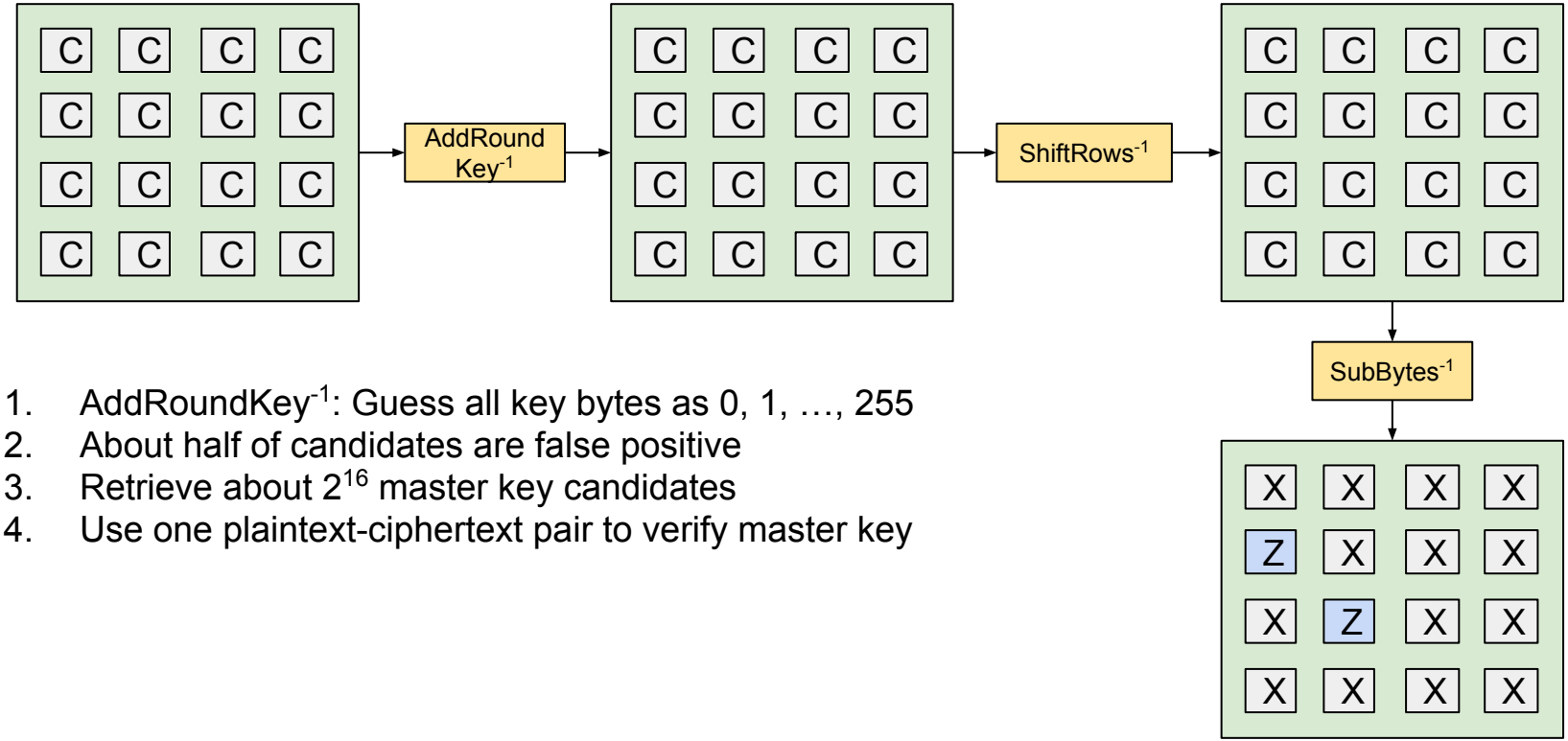
$\oplus$

....

=

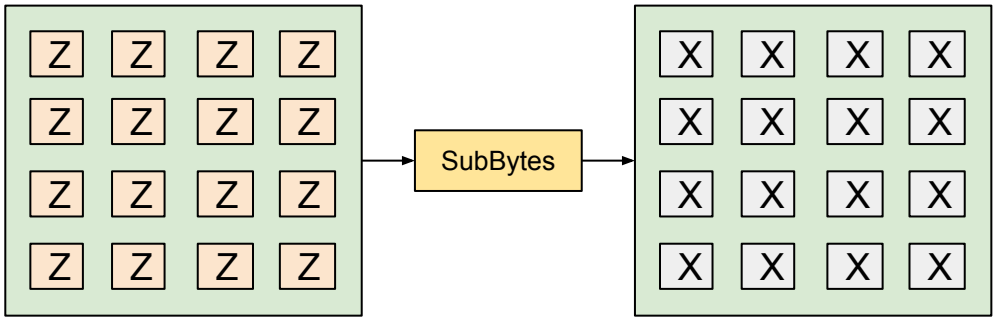
$S = 0 \oplus 0 \oplus 0 \oplus 0 = \mathbf{0}$

Key byte guessing



1. AddRoundKey^{-1}: Guess all key bytes as 0, 1, …, 255
2. About half of candidates are false positive
3. Retrieve about 2^{16} master key candidates
4. Use one plaintext-ciphertext pair to verify master key

Round 4



exercise

Drive: crypto4/square/

aes.py: aes implementation, bonus method: master_from_round -> calculate master key from round key

client.py:

encrypt(data) -> returns encrypted data (one block)

get_encrypted_flag() -> returns encrypted flag

goal: **decrypt flag**

Pseudorandom generators

PRNG vs CSPRNG

Self explanatory:

Pseudo **R**andom **N**umber **G**enerator

Cryptographically **S**ecure **P**seudo **R**andom **N**umber **G**enerator

PRNG also known as a **D**eterministic **R**andom **B**it **G**enerator

PRNGs

Given

- P – a probability distribution on $(\mathbb{R}, \mathfrak{B})$ (where $\mathfrak{B}$ is the standard Borel field on the real line)
- $\mathfrak{F}$ – a non-empty collection of Borel sets $\mathfrak{F} \subseteq \mathfrak{B}$, e.g. $\mathfrak{F} = \{(-\infty, t] : t \in \mathbb{R}\}$. If $\mathfrak{F}$ is not specified, it may be either $\mathfrak{B}$ or $\{(-\infty, t] : t \in \mathbb{R}\}$, depending on context.
- $A \subseteq \mathbb{R}$ – a non-empty set (not necessarily a Borel set). Often A is a set between P's support and its interior; for instance, if P is the uniform distribution on the interval $(0, 1]$, A might be $(0, 1]$. If A is not specified, it is assumed to be some set contained in the support of P and containing its interior, depending on context.

We call a function $f : \mathbb{N}_1 \to \mathbb{R}$ (where $\mathbb{N}_1 = \{1, 2, 3, \ldots\}$ is the set of positive integers) a **pseudo-random number generator for P given $\mathfrak{F}$ taking values in A** iff

- $f(\mathbb{N}_1) \subseteq A$
- $\forall E \in \mathfrak{F} \quad \forall 0 < \varepsilon \in \mathbb{R} \quad \exists N \in \mathbb{N}_1 \quad \forall N \leq n \in \mathbb{N}_1, \quad \left| \frac{\#\{i \in \{1, 2, \ldots, n\} : f(i) \in E\}}{n} - P(E) \right| < \varepsilon$

($\#S$ denotes the number of elements in the finite set S.)

It can be shown that if f is a pseudo-random number generator for the uniform distribution on $(0, 1)$ and if F is the CDF of some given probability distribution P, then $F^* \circ f$ is a pseudo-random number generator for P, where $F^* : (0, 1) \to \mathbb{R}$ is the percentile of P, i.e. $F^*(x) := \inf\{t \in \mathbb{R} : x \leq F(t)\}$. Intuitively, an arbitrary distribution can be simulated from a simulation of the standard uniform distribution.

PRNGs

- *"Algorithm for generating a sequence of numbers whose properties* ***approximate*** *the properties of* ***sequences of random numbers****"*

- Perfect PRNG = indistinguishable from random sequence

- Trivia: with N bits of internal state, PRNG will repeat itself every 2**N steps.

- PRNG test: DieHard, DieHarder. For example:
  - The same number of 1s and 0s
  - Distribution of ngrams
  - Spacings between random points
  - Computing pi

PRNGs: general form

```
class prng_generic:
    def __init__(self, seed):
        self.state = self.initial_state(seed)

    def next(self):
        self.update_state()
        return self.current()
```

PRNGs: middle-square

- middle-square method: one of first methods of generating pseudorandom numbers
- Invented and used by John von Neumann

- **1234567890** **2
  1524**15787501905**2100

  next(**1234567890) = 1578750190**

PRNGs: middle-square

```
class prng_middle_square:
    def __init__(self, seed):
        self.state = seed

    def next(self):
        m = '0'*10 + str(self.state**2)
        self.state = int(m[-20:][5:15])
        return self.state
```

PRNGs: LCG

- **L**inear **C**ongruential **G**enerator
- One of oldest and most popular PRNGs
- Fast, very easy to implement
- Used by old Visual Studio C++, Visual Basic <= 6, Delphi, Pascal, glibc (sometimes), Borland C++, java.util.Random

- `state = (state*m + c) % n`

  next(**1234567890) = 804752948**
  (with m=1103515245, c=12345, n=2**31-1=2147483647)

PRNGs: LCG

```
class prng_lcg:
    def __init__(self, seed):
        self.state = seed

    def next(self):
        self.state = (self.state * m + c) % n
        return self.state
```

PRNGs: Putting *pseudo* back in *P*RNG

https://var.tailcall.net/prng/lcg1_easy

PRNGs: Mersenne Twister

- **The** most popular PRNG?
- Fast, rather simple
- Standard implementation is based on $2^{19937}-1$
- Used by Python, Ruby, PHP, Visual Studio C++, GMP, Common Lisp, Free Pascal, Sage, Excel...
- https://en.wikipedia.org/wiki/Mersenne_Twister#Python_implementation

Linear Feedback Shift Registers

```
state = (state << 13) ^ state
```

- Useful for pseudo-random numbers generation
- Reversible transformation
- Used in Mersenne Twister

PRNGs: Mersenne Twister

```
class prng_mt:
    def __init__(self, seed):
        self.index = 624
        self.mt = [0] * 624
        self.mt[0] = seed
        for i in range(1, 624):
            self.mt[i] = 0xffffffff & (1812433253 *
               (self.mt[i - 1] ^ self.mt[i - 1] >> 30) + i)
```

(code ~~stolen~~borrowed from wikipedia)

PRNGs: Mersenne Twister

```
def next(self):
    if self.index >= 624:
        self.twist()

    y = self.mt[self.index]
    y = y ^ y >> 11
    y = y ^ y << 7 & 2636928640
    y = y ^ y << 15 & 4022730752
    y = y ^ y >> 18
    self.index = self.index + 1
    return y & 0xFFFFFFFF
```

PRNGs: Mersenne Twister

```
def twist(self):
    for i in range(624):
        y = 0xFFFFFFFF & ((self.mt[i] & 0x80000000) +
                          (self.mt[(i + 1) % 624] & 0x7fffffff))
        self.mt[i] = self.mt[(i + 397) % 624] ^ y >> 1

        if y % 2 != 0:
            self.mt[i] = self.mt[i] ^ 0x9908b0df
    self.index = 0
```

PRNGs: Putting *pseudo* back in *P*RNG

https://var.tailcall.net/prng/lfsr1_first

CSPRNGs

Like PRNGs... But **C**ryptographically **S**ecure

Next Bit Test:
Given k bits of output, adversary can't predict next bit with probability > 50%

State Compromise Extension Resistance:
After state exposed, impossible to predict previous numbers.

PRNG vs **CS**PRNG

	Next Bit Test	State Extension
LCG	✗	✗
Mersenne Twister	✗	✗

CSPRNG design

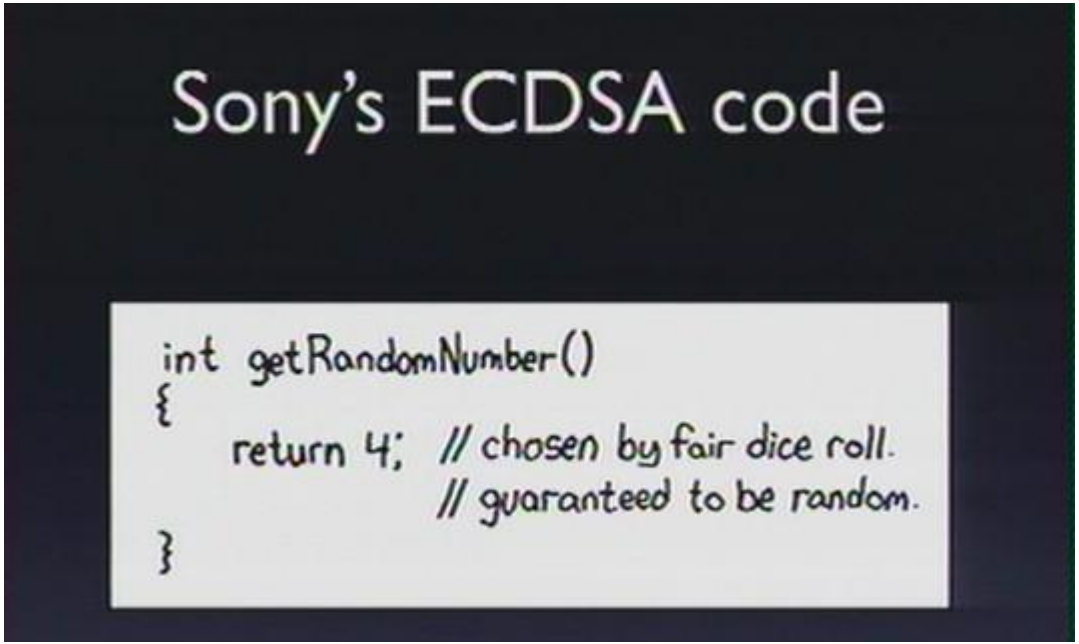
Designed "from the ground up":

- Blum Blum Shub (quadratic residuosity problem)
- Blum-Micali (discrete logarithm problem)

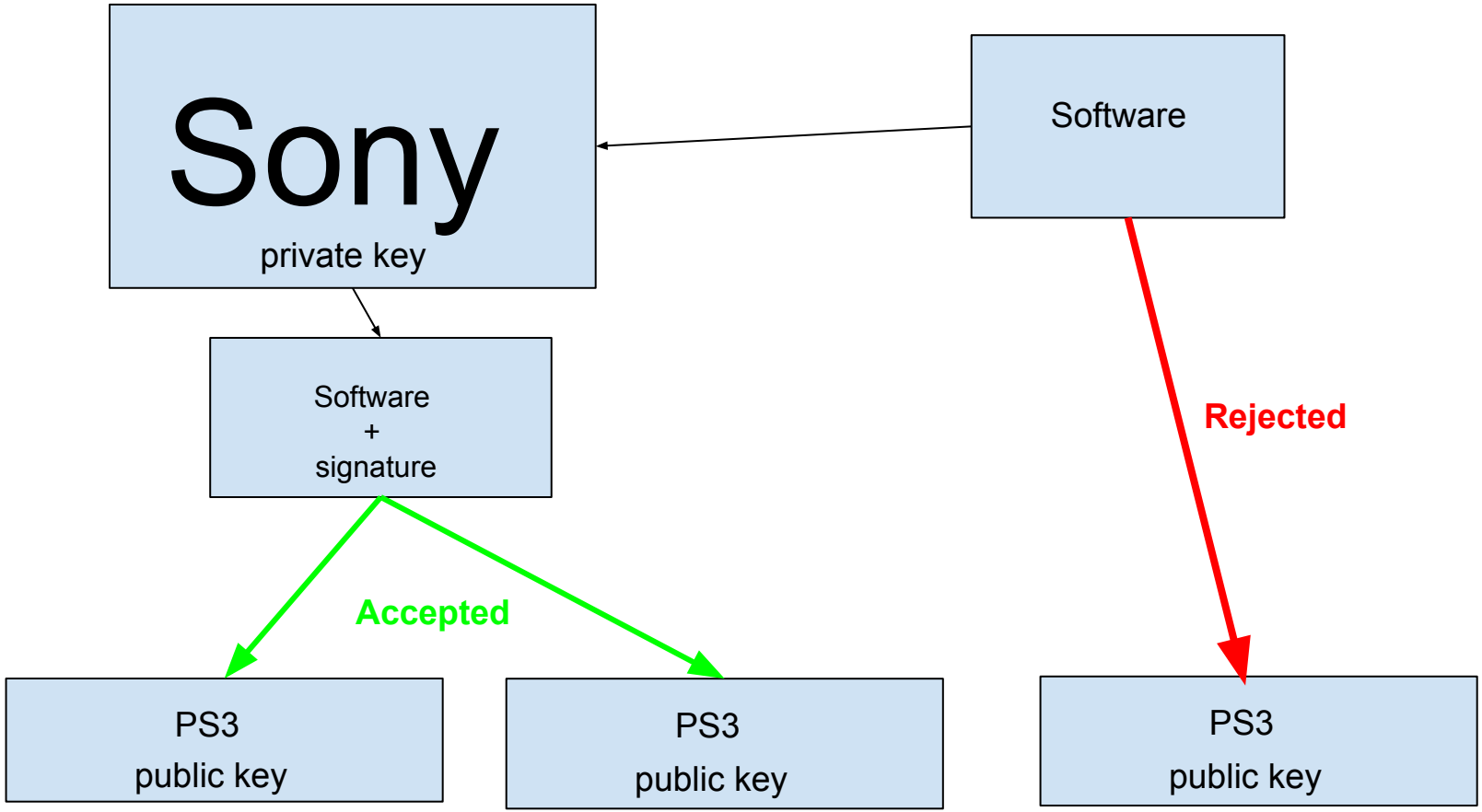
Based on crypto primitives:

- Block Cipher → CSPRNG
- Stream Cipher → CSPRNG
- Secure Hash → CSPRNG

Sony crypto fail



- Team fail0verflow
- Completely broken Playstation 3 console
- Recovered private key which wasn’t present on device
- Bug in Sony ECDSA signing implementation


Sony
private key
Software
Software
+
signature
Rejected
Accepted
PS3
public key
PS3
public key
PS3
public key

DSA algorithm

Parameters:
Numbers P, Q, G
Hash Function H

Private Key:
random X: $0 < X < Q$

Public Key:
Y: $Y = G^X \bmod P$

Singing: M - message
Choose random K: $0 < K < Q$
Calc: R: $R = (G^K \bmod P) \bmod Q$
Calc: S: $S = K^{-1}(H(M)+X*R) \bmod Q$
Signature: (R, S)

Verification: M - message, (R, S) - signature
$W = S^{-1} \bmod Q$
$V = (G^{H(M)*W \bmod Q} * Y^{R*W \bmod Q} \bmod P) \bmod Q$
Correct if V = R

Repeated K

- Two signed messages: M, M’
- Signatures: (R, S), (R’, S’)
- K = K’
- R = R’
- $S - S' = (K^{-1}(H(M)+X*R) - K^{-1}(H(M')+X*R)) \bmod Q = K^{-1} * (H(M) - H(M')) \bmod Q$
- $K = (H(M) - H(M')) * (S - S')^{-1} \bmod Q$
- **X** $= (S * K - H(M)) * R^{-1} \bmod Q$

Singing: M - message
Choose random K: 0 < K < Q
Calc: R: $R = (G^K \bmod P) \bmod Q$
Calc: S: $S = K^{-1}(H(M)+X*R) \bmod Q$
Signature: (R, S)

exercise

Drive: crypto4/dsa/

client.py:

verify(data, signature) - check if signature is correct for data

verify(“flag”, signature) - if signature is correct then flag is returned

sigs.txt:

msg, signature pairs for 50 messages generated by gen.py

Bibliography

Joan Daemen and Vincent Rijmen, "AES Proposal: Rijndael"

FIPS PUB 186-4: Digital Signature Standard (DSS), July 2013